Fibonacci and His Sequence

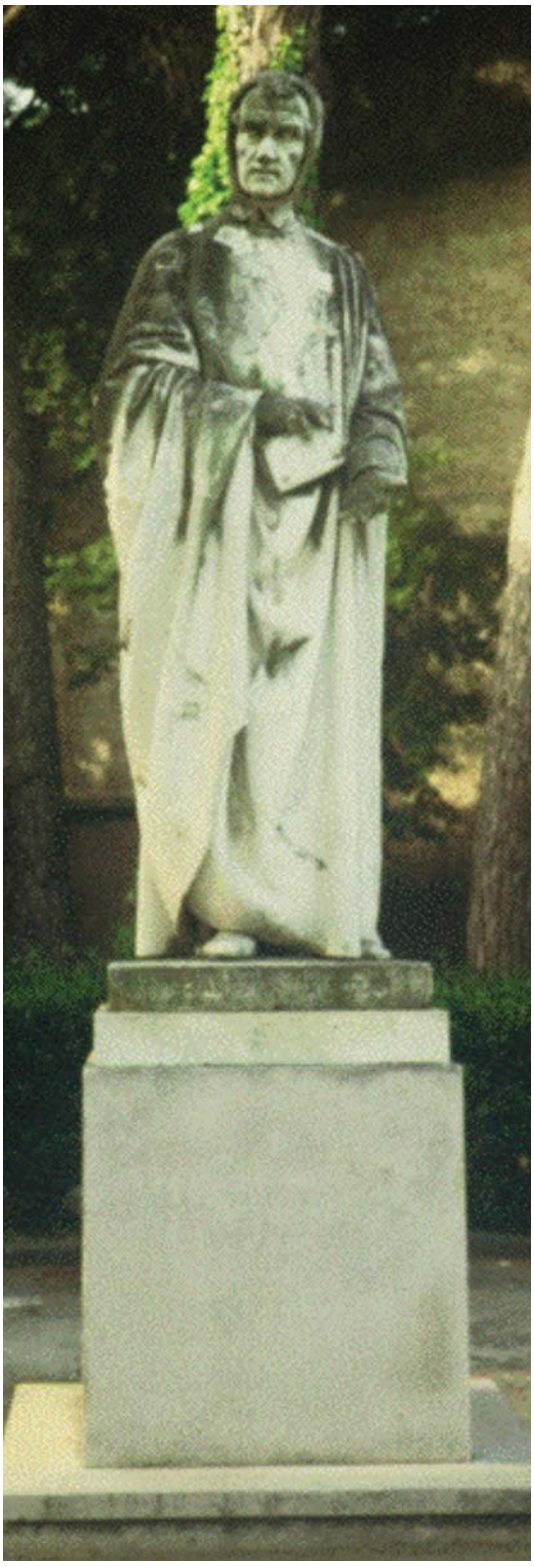
by Rose Williams

Leonardo of Pisa or Leonardo Fibonacci (c. 1170-c. 1240) has been called the Father of the mathematical Renaissance as Petrarch is the Father of the literary Renaissance. As his father was an overseer in a Pisan mercantile house in Bugia in North Africa, Fibonacci spent much of his youth there. He learned from Muslim teachers and became enamored of the Hindu-Arabic numeral system, both because of its practical value in business and because of its infinite possibilities for pure mathematics. His largest and most important work, the **LIBER ABBACI**, thoroughly and systematically explains the Hindu-Arabic numeral system (which Fibonacci calls the nine figures of the Indians), the use of zero, and the decimal system to the Christian West. Durant says that only a few mathematicians realized that the symbols and decimal alignments he had brought west opened the way to mathematical developments impossible before.

He was the first Westerner to employ arithmetic, geometry, and algebra together to reach mathematical solutions not feasible in only one of them. Much of his work delves deep into mathematical theory and is outside the scope of our study, but it made incalculable contributions to the modern world. Eves calls him "the most talented mathematician of the Middle Ages."

Among the problems given to help students practice the use of the Hindu-Arabic numerals was a theoretical one concerning the breeding of rabbits. Its solution produced the number sequence known as the Fibonacci series, in which each number is the sum of the previous two and, if divided by the next highest number, gives a ratio of golden proportion. This series and its ratio appear extensively both in nature and in man-made artifacts.

Asimov says that Fibonacci's book gave the deathblow to the old system of numbering by the alphabet. To appreciate Fibonacci, we have only to try to divide MCMLVIII by LXVI.



Statue of Fibonacci in Pisa

Bibiographical References:

1. Asimov, Isaac. *Bibliographical Encyclopedia of Science and Technology.* New York: Avon, 1976.
2. Durant, Will. *The Age of Faith.* Simon and Schuster, 1980.
3. Eves, Howard. *An Introduction to the History of Mathematics.* Holt, Rinehart & Winston. 1976.

Pisa Baptistry - On of the many geometrical buildings of Fibonacci's home city

Main Text, Section I

LIBER ABBACI introduction

(Unusual words are in Italics; these are explained in the vocabulary, which is for both sections of text)

Incipit Liber Abaci compositus a Leonardo filio Bonacii Pisano in anno MCCII

Scripsistis mihi domine, mi magister Michael Scotte, summe philosophe, ut librum de numero, quem dudum composui,vobis transcriberem: unde vestrae obsecundans postulationi, ipsum subtiliori perscrutans Indagine ad vestrum honorem et aliorum multorum utilitatem correxi. In cuius correctione qu(a)edam necessaria addidi, et qu(a)edam superflua resecavi. In quo plenam numerorum *doctrinam* edidi, iuxta modum Indorum, quem modum in ipsa scientia prestantiorem elegi. Et que arismetrica et geometria scientia sunt connexe, et *suffragatorie* sibi ad invicem, non potest de numero plena tradi doctrina, nisi *intersecantur* geometrica quedam, vel ad geonetriam *spectantia*, que hic tantum iuxta modum numeri operantur; qui modus est sumptus ex multis probationibus et demonstrationibus, que figuris geometricis fiunt. Verum in alio libro, quem de practica Geometrie composui, ea que ad Geometriam pertinent et alia plura copiosis explicavi, singula *subiectis* approbationibus geometricis demonstrando. Sane hic liber magis ad theoricam spectat quam ad praticam. Unde qui per eum huius scientie practicam bene scire voluerint, oportet eos continue usu et exercition diuturno in eius praticis perstudere: quod scientia per practicam versa in habitum, memoria et intellectus adeo *concordent* cum manibus et figuris, quod quasi uno impulsu in uno et eodem *instanti* circa idem per omnia naturaliter *consonent:* et tunc cum fuerit discipulus habitudinem consecutus, gradatim poterit ad perfectionem huius facile pervenire.

Cum genitor meus a patria publicus scriba in duana Bugee pro Pisanis mercatoribus ad eam confluentibus constititus preesset, me in pueritia mea ad se venire faciens, inspecta utilitate et *commoditate*

futura, ibi me studio abbaci per aliquot dies stare voluit et doceri. Ubi ex mirabili magisterio in arte per novem figuras Indorum introductus, scientia artis in tantum mihi pre ceteris placuit, et intellexi ad illam, quod quicquid studebatur ex ea apud Egyptum, Syriam, Greciam, Siciliam, et proviniciam cum suis variis modis, ad que loca negotiationis tam postea peragravi per multum studium et disputationis didici conflictum. Sed hoc totum etiam et *algorismum* atque arcus pictagore quasi errorem computavi respectu modi Indorum. Quare amplectens strictius ipsum modum Indorum, et attentius studens in eo, ex proprio sensu quedam addens, et quedam etiam ex *subtilitatibus* Euclidis geometrice artis *apponens,* summam huius libri, quam intelligibilius potui, in xv capitulis distinctam componere laboravi, fere omnia que *inserui,* certa probatione ostendens, ut extra,perfecto pre ceteris modo, hanc scientiam appetentes instruantur, et gens Latina de cetero, sicut hactenus, *absque* illa minime inveniatur. Si quid forte minus aut plus iusto vel necessario intermisi, mihi deprecor indulgeatur; cum nemo sit qui vitio careat, et in omnibus undique sit circumspectus.

Vocabulary

1. **doctrina, doctrinae** *f* teaching, system of knowledge
2. **suffragator, suffragatoris** *m* supporter
3. **interseco** (1) involve
4. **subicio, subicere, subieci, subiectum** substitute
5. **concordo** (l) to be in harmony
6. **instans, instantis** *n* present moment
7. **consono, consonare, consonui_**-- re-echo,sound together
8. **commoditas, commoditatis** *f* convenience
9. **algorismum, algorismi** *n* calculation (a word coined from the name of an Arabian mathematician)
10. **subtilitas, subtilitatis** *f* exactness
11. **appono, apponere, apposui, appositum** reckon, put beside
12. **insero, inserere, inserui, insertum** introduce, insert
13. **absque** *prep (w abl)* without, but for
14. **par, paris** *n* pair
15. **pario, parere, peperi, par(i)tum** to bear, produce

Comprehension Questions

1. Why does Fibonacci say he has involved geometric principles in the teaching of arithmetic?
2. How does he say students may become proficient at math?
3. What did Fibonacci discover through his travels that he wants to share with the West?
4. How does he conclude this introduction?

Grammar/ Word Use Questions

1. Nouns are constantly being made from verbs or verb parts; INSTANS is a substantive made from a present participle. Translate it in accordance with the sense of the passage.
2. After SI, NISI, & NE the forms of QUIS, QUID may be translated “anyone” or

“anything”. Translate SI QUID FORTE INTERMISI.

3. List and transform variations in spelling: PRATICUM/PRACTICUM; QUE/QUAE

Main Text, Section II

Quot Paria Cuniculorum

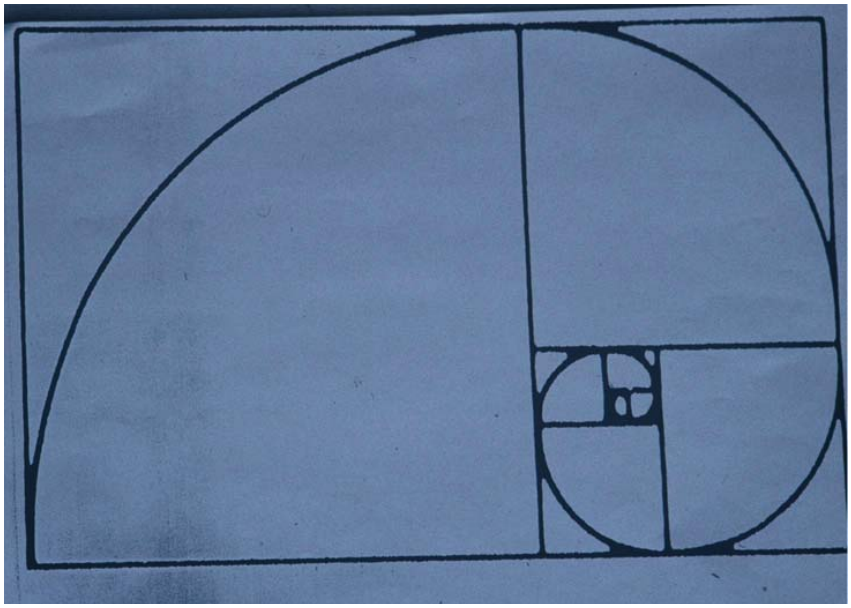
Quot paria cuniculorum in uno anno ex uno pari germinentur. Quidam posuit unum *par* cuniculorum in quodam loco, qui erat undique pariete circumdatus, ut sciret, quot ex eo paria germinarentur in uno anno: cum natura eorum sit per singulum mensem aliud par germinare; et in secundo mense ab eorum navitate germinant. Quia suprascriptum par in primo mense germinat, duplicabis ipsum, erunt paria duo in uno mense. Ex quibus unum, scilicet primum, in decundo mense geminat; et sic sunt in secundo mense paria 3; ex quibus in uno mense duo pregnantur; et geminantur in tercio mense paria 2 cuniculorum; et sic sunt paria 5 in ipso mense; ex quibus in ipso pregnantur paria 3; et sunt in quarto mense paria 8; ex quibus paria 5 geminant alia paria 5: quibus additis cum pariis 8, faciunt paria 13 in quinto mense; ex quibus paria 5, que geminata fuerunt in ipso mense, non concipiunt in ipso mense, sed alia 8 paria pregnantur; et sic sunt in sexto mense paria 21; cum quibus additis pariis 13, que geminantur in septimo, erunt in ipso paria 34; cum quibus additis pariis 21, que geminantur in octavo mense, erunt in ipso paria 55; cum quibus additis pariis 34, que geminantur in nono mense, erunt in ipso paria 89; cum quibus additis rursum pariis 55. que geminantur in decimo, erunt in ipso paria 144; cum quibus additis rursum pariis 89, que geminantur in undecimo mense, erunt in ipso paria 233. Cum quibus etiam additis pariis 144, que geminantur in ultimo mense, erunt paria 377; et tot paria *peperit* suprascriptum par in prefato loco in capite unius anni. Potes enim videre in hac margine, qualiter hoc operati fuimus, scilicet quod iunximus primum numerum cum secundo videlicet 1 cum 2; et secundum cum tercio; et tercium cum quarto; et quartum cum quinto, et sic deinceps, donec iunximus decimum cum undecimo, videlicet 144 cum 233; et habuimus suprascriptorum cuniculorum summam videlicet 377; et sic posses facere per ordinem de infinitis numeris mensibus.

Comprehension Questions

1. Like the other math problems in his book, Fibonacci's famous "rabbit problem" was designed to give students practice in the new system of 9 numbers as opposed to the Roman numerals. It is theoretical, not naturalistic, as Fibonacci sets limits on such factors as how many young each pair of rabbits will have and what the genders of the young will be. Make a chart of the twelve months and how many pairs there will be each month.

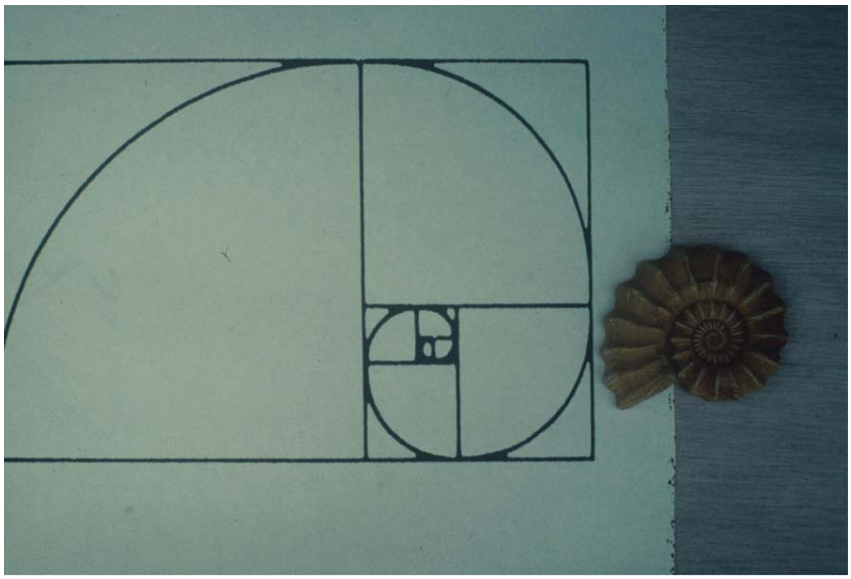
2. Apparently Fibonacci created this problem because he was fascinated by this numerical progression obtained by beginning with 1 and simply adding each number to the next. This series of numbers in relationship to each other produce a "golden proportion"found extensively in nature and in art. Divide 2 by 3; then 3 by 5, then 5 by 8. How close are your answers to .618? Are they drawing nearer to it?

3. Draw a rectangle whose sides are 13 centimeters and 21 centimeters. This is an approximate "golden rectangle" whose sides have a ratio close to .618. Structures designed to please the human eye, from the Parthenon to playing cards, have this form. Now draw a line cutting the figure into a square with sides of 13 centimeters with a smaller rectangle on the side (See Photo). Keep repeating this process until the squares are too small to draw. This is the rectangle of the "whirling squares".

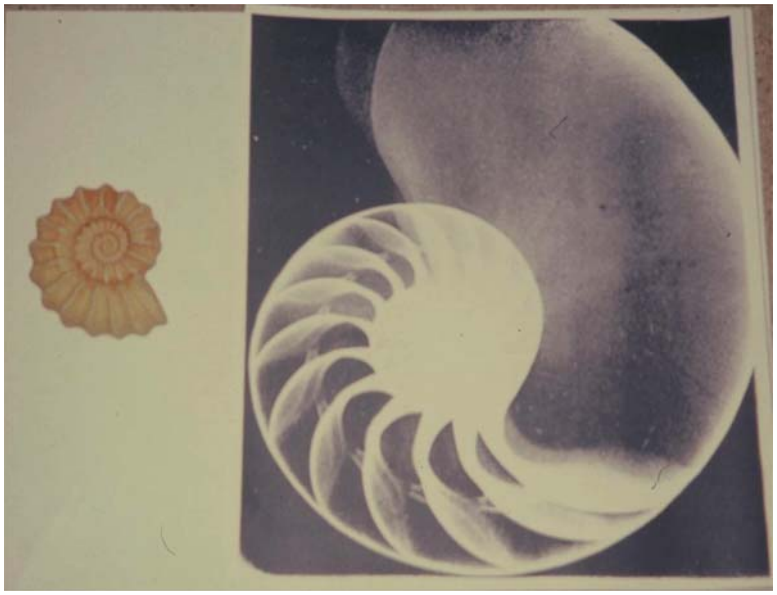


The Whirling Squares

4. Draw a 90 degree arc connecting opposite corners of each square. This is the golden spiral which is found in ram's horns, umbilical cords, elephant tusks, shells of sea creatures, and countless other natural structures which need to grow without changing essential shape.



Whirling Squares and shells



Spiraling shells

5. Plants as well as animals have growth patterns regulated by Fibonacci numbers. These are easiest to see on pine cones, artichokes, and pineapples. Pine cones and artichokes typically have 3, 5, 8, or even 13 rows of bracts or scales spiraling gradually in direction, and a larger number spiralling steeply in an opposing direction. (Each bract is a member of two rows, of course). Examine the picture below for evidence of this.

Pine Cone with spiraling bracts

Grammar/Word Use Questions

1. We have seen throughout our study that words may change in usage and meaning through the passage of time or as authors adapt language to their thoughts. GERMINO properly means "to sprout", and GEMINO "to double". How from the context can they best be translated in this passage?

2. The vocabulary lists the noun PAR, PARIS for "pair". What evidence do you see that Fibonacci also uses a doublet noun PARIUM, PARII for "pair"?

Teachers Notes for Fibonacci

Like many life-long classicists, I approach math lessons like this one with fear and trembling. As it is an important part of our cultural history, and of the contribution of the Latin language to the modern world, I decided to include it. You and your students might want to consult math people on this lesson, as I have done. I have learned many fascinating things about human perception while studying it.

Translation

Begins the Book of the Abacus composed by Leonardo filio Bonacii Pisano in the year 1202.

You have written to me, master, my teacher Michael Scott, greatest of philsophers, so that I might transcribe for you my book about number which I composed not long ago: whence complying with your claims and searching with more precise inquiry I corrected it itself to your honor and the usefulness of many. In which correction I added certain necessary things, and cut out certain superfluous ones. In which I added the complete teaching of numbers like the method of the Indians, which method I chose as more excellent in the learning itself. And as the arithmetic and geometric disciplines are connected and are supporters of each other in turn, the complete teaching about numbers is not able to be handed down unless certain geometric principles or principles looking to geometry (are) involved which operate only joined to this method of number; which method is taken from many proofs and demonstrations which are made with geometric figures. Indeed in another book which I composed about the practice of Geometry, those things which pertain to Geometry and other things more fully I explained by demonstrating one at a time with geometric proofs substituted. Of course this book looks more to theory than to practice. Therefore (for) those who want to know the practice of this learning well through it, it is necessary to study continually by use and daily exercise in the practice of it; because knowledge having been turned into habit through practice, the memory and the mind therefore may be in harmony with the hands and the figures because they reecho as if by one impulse in one and the same moment around the same through all things naturally; and then since the student will have followed the habit little by little he will be able to arrive easily at the perfection of this.

When my father was sent as a public scribe from our native city and established in customs duty at Bugia for Pisan merchants meeting at that city, directing me in my boyhood to come to him, having considered future usefulness and convenience, there he wanted me to be taught and to continue in the study of the abacus through many days. When having been introduced from wonderful teaching into the art through the nine figures of the Indians, the knowledge of this art was greatly pleasing to me above the rest, and I understood with regard to that, because whatever was studied through it in Egypt, Syria, Greece, Sicily, and the province with their own various methods, to those places of business afterward I traveled and learned through much study and contrasting of discussion. But this whole (thing) also both the Arabic calculation and arcs of the pictographer as if an error I computed with consideration of the method of the Indians. Therefore embracing more strictly the method of the Indians itself and more attentively studying in it, adding certain things from a proper sense and also reckoning certain things from the exactness of the geometric art of Euclid, the sum total of this book which I was able to make more understandable, I labored to expound distinctly in 15 chapters, generally all things which I inserted showing by certain proof, and beyond that, this method having been perfected before the rest, those seeking are taught this knowledge and the Latin kind in accordance with the rest, just as up

til now without that it may be found very seldom. If anything more or less necessary by chance I have omitted, I pray it may be tolerated for my sake, since no man may live who may lack faults, and in all things on all sides may be cautious.

From PROBLEMS

How many pairs of rabbits in one year may be bred from one pair.

A certain man placed one pair of rabbits in a certain place, which was surrounded on all sides by a wall so that he might know how many pairs from this one might be bred in one year: since the nature of these is in a single month to breed another pair, and in the second month from their birth they breed. Because the above mentioned pair breeds in the first month, you will duplicate it; there will be two pairs in one month. From which one just as the first will breed in the second month; and so there are in the second month 3 pairs; from which in one month two are pregnant; and in the third month two pairs of rabbits are born; and so there are 5 pairs in this month; from which in this month 3 pairs are pregnant; and there are in the fourth month 8 pairs; from which 5 pairs bear another 5 pairs; these having been added with the 8 pairs, 13 pairs are made in the fifth month; from which 5 pairs, which were born in the very month, do not conceive in the same month, but the other 8 pairs are pregnant; and so there are in the sixth month 21 pairs, with these 13 pairs added, which are born in the seventh month, there are 34 pairs in it; with the 21 pairs added, which are born in the eighth month, there will be 55 pairs in it; with those 34 pairs added which are born in the ninth month there will be 89 pairs in it; with those 55 pairs added again which were born in the tenth month there will be 144 pairs; with those 89 pairs added which were born in the eleventh month there will be 233 pairs in those one. With those 144 pairs also added which are born in the last month there will be 377 pairs; and so many pairs the one pair written above produced in a prepared place in one year. For you are able to see in this framework how we have worked this; the fact that of course we joined the first number with the second, that is, 1 with 2; and the second with the third; and the third with the fourth; and the fourth with the fifth, and so successively until we joined the tenth with the eleventh, that is, 144 with 233, and we have the total of the rabbits written above, of course 377, and so you are able to do through the order of an infinite number of months.

Series of pairs:

Parium 1 primus 2 secundus 3 tercius 5 quartus 8
quintus 13 sestus 21 septimus 34 octavus 55 nonus 89
decimus 144 undecimus 233 duodecimus 377